NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL MEMORANDUM 1381

ON THE DETERMINATION OF CERTAIN BASIC TYPES OF

SUPERSONIC FLOW FIELDS

By Carlo Ferrari

Translation of "Sulla determinazione di alcuni tipi di campi di
corrente ipersonora," Rendiconti dell'Accademia
Nazionale dei Lincei, Classe di Scienze fisiche,
matematiche e naturali, serie VIII, vol. VII,
no. 6; read at the meeting held on
December 10, 1949.

NACA

Washington
November 1954

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1381

ON THE DETERMINATION OF CERTAIN BASIC TYPES OF

SUPERSONIC FLOW FIELDS*

By Carlo Ferrari

SUMMARY

A quite universal mode of attack on problems which arise in super-sonic flow, whether connected with flow over wings or over bodies of revolution, is explained, first, in great generality, and then in more detail, as specific applications to concrete cases are illustrated. The method depends on the use of Fourier series in the formal definition of the potential governing the flow and in the setting up of the boundary conditions. This new formulation of the many problems met in supersonic flow is really an extension of the doublet type of "fundamental solution" to higher order types of singularity. The limitations and, in contrast, the wide field of applicability of such a means of handling these problems with complex boundary conditions is discussed in some detail, and a specific example of a wing-body interference problem is cited as proof of the versatility of the method, because the results obtained by applying the techniques expounded herein agree well with experimentally determined data, even for the quite complex configuration used to exemplify the kind of problem amenable to such treatment.

1. INTRODUCTION

For purposes of analytic treatment of the flow problem to be con-sidered here the usual rectangular Cartesian coordinate system is employed with the x-axis taken to lie in the direction of and having the same sense as the uniform (undisturbed) free-stream velocity, $\overrightarrow{V_\infty}$. This free-stream velocity, $\overrightarrow{V_\infty}$, is taken to be supersonic in the discussion that follows; i.e., $V_\infty > C_\infty$ where C_∞ denotes the velocity of sound in the undis-turbed stream. The flow of the gaseous fluid to be investigated is to

---

*"Sulla determinazione di alcuni tipi di campi di corrente iper-sonora," Rendiconti dell'Accademia Nazionale dei Lincei, Classe di Scienze fisiche, matematiche e naturali, serie VIII, vol. VII, no. 6; read at the meeting held on December 10, 1949.

be considered as resulting from the superposition upon the free-stream
velocity of a nonuniform flow, having velocity components that are des-
ignated as V_x, V_y, and V_z, and lying in the direction of the respec-
tive axes (x,y,z) of the coordinate system. This nonuniform super-
imposed flow is supposed to be small enough, in comparison with the
speed of sound, C_∞, that it is permissible to neglect the ratios V_x/C_∞,
V_y/C_∞, etc. in the equations governing the flow.

It is taken for granted that, under the conditions stated above,
there exists a velocity potential describing the flow in question, and
in practically all cases which are of any interest for actual designs
it will really be true that this assumption can be made legitimately.
If it is then agreed that the nonuniform superimposed part of the flow
is to be denoted by the potential ϕ, it will be recognized that this
potential will have to satisfy the relationship:

$$\left(1 - M_\infty^2\right) \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{1}$$

where the free-stream Mach number, M_∞, is defined as $M_\infty = V_\infty/C_\infty$ and,
of course, the potential ϕ will also have to obey the boundary condi-
tions which are peculiar to each stated problem.

A way of handling the determination of the function ϕ, so that it
will satisfy equation (1) and so that it will obey the imposed boundary
conditions, will now be explained, and its usefulness illustrated by
consideration of problems which can be attacked by this means, both in
the case of lifting surfaces (that is, wings) as well as in the case of
bodies of revolution. The proposed method is based on the use of Fourier
series. Although this technique does not afford complete universality
in treatment of all the posed problems, as will be more clearly pointed
out in what follows, it can be used to fine advantage in a goodly number
of situations by replacing the procedures which are based on the Fourier
or the Laplace transforms (which, for that matter, have just as restricted
limits of applicability as the analogous ones which arise in connection
with the approach being discussed herein) or by being substituted in
place of the techniques which stem from use of the "fundamental" (source,
sink, doublet) solutions to equation (1), or from use of transformations
carried out in the complex plane.

2. PROBLEMS HAVING TO DO WITH FLOW OVER WINGS

As usual, the wings are imagined to be very slender and so placed
that the wing span lies along the y-axis; i.e., the long dimension is

out the y-axis (see fig. 1). Let the equations which define the ventral and dorsal surfaces of the wing surface be, in fact, given in the form:

$$z = z_v(x,y) \quad \text{and} \quad z = z_d(x,y)$$

and then the slenderness of the wing is supposed to be slight enough that the above-defined values of z will be so small at all locations on these surfaces as to make it possible to accept the fact that the derivative $\dfrac{\partial z}{\partial x}$ is, to all intents and purposes, equal to the direction cosine, with respect to the free-stream x-axis, of the normal to the surface. It is further assumed that the wing is immersed in a stream of supersonic flow which has a constant value for its component lying in the direction of the x-axis, of magnitude V_∞. The component in the direction of the z-axis, meanwhile, is assumed to be known, but of relatively small size in comparison with the V_∞ velocity, and it may take on various values, which will be denoted by $V_z{}'$. If, now, the potential describing the flow perturbed by the wing is denoted by ϕ this potential will have to satisfy equation (1), and it will also have to conform to the conditions which are imposed at the boundaries. These further (boundary) conditions may be stated as follows:

(2) Upstream of a certain surface, which may be immediately defined just as soon as the wing-like body is specified which is to invest the impinging stream, the value of ϕ is zero; i.e., the basic condition is

$$\phi = 0 \qquad\qquad (2)$$

(3) On the wing surface, it must be true that

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = -V_\infty \frac{\partial z}{\partial x} - V_z{}' \, \cos\left(\vec{n},\vec{z}\right) = H(x,y) \qquad (3)$$

wherein the value of z to be employed is either the z_v or z_d quantities, depending on whether one is concerned with a point which is lying on the under ventral surface or on the upper dorsal surface, respectively. The notation $\cos\left(\vec{n},\vec{z}\right)$ signifies the cosine of the angle between the z-axis and the unit vector taken in the direction of the exterior normal to the wing surface in question; i.e., this vector is represented by the vector n, and under the present hypothesis $\cos\left(\vec{n},\vec{z}\right) = \pm 1$.

It is convenient to distinguish between two basic types of problem which come under this kind of analysis, and to make the differentiation on the basis of the sort of boundary conditions met with in each type; that is,

Symmetric Types of Configuration

In this case, the boundary conditions to be satisfied on the wing may be expressed in the form

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0^+} = H(x,y) \tag{3'}$$

and

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0^-} = -H(x,y)$$

Asymmetric Types of Configuration

In this case, the boundary conditions are expressed as

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0^+} = \left(\frac{\partial \phi}{\partial z}\right)_{z=0^-} = G(x,y) \tag{3''}$$

The first type of problem corresponds to a configuration for which the wing has a zero angle of attack with respect to the free-stream undisturbed flow, $\vec{V}_\infty$, and which possesses a symmetric profile. The second type of problem corresponds to a configuration for which the wing is a flat plate, but which has any local angle of attack whatsoever, with respect to the free-stream vector, $\vec{V}_\infty$, so long as it is small.

3. DEVELOPMENT OF THE CASE OF THE SYMMETRIC TYPE OF CONFIGURATION

In this case it will suffice to examine the flow solely in the upper half-plane, where $z > 0$. If $\phi^{(1)}(x,y,z)$ stands for the flow which takes place in this upper region, and if $\phi^{(2)}(x,y,z)$ represents the flow in the nether region, then, of course,

$$\phi^{(2)}(x,y,z) = \phi^{(1)}(x,y,-z)$$

The boundary conditions in this case are composed of equations (3'), together with the restriction that

$$\left(\frac{\partial \phi^{(1)}}{\partial z}\right)_{z=0} = 0 \quad \begin{array}{l}\text{(for locations lying beyond the region} \\ \text{occupied by the wing surface)}\end{array} \qquad (2')$$

Now let the definition of the function describing the velocity component at the wing surface, and also the potential function itself, be cast into the convenient forms

$$H^*(x,y) = V_\infty \sum_m H_m \left(\frac{x}{b}\right) \cos \frac{2\pi m y}{b} = V_\infty \sum_m H_m(\xi) \cos \frac{\pi}{2} m \eta$$

and

$$\phi^{(1)}(x,y,z) = \phi^{(1)} = V_\infty \, b \sum_m \phi_m(\xi,\zeta) \cos \frac{\pi}{2} m \eta \qquad (4)$$

wherein $\xi = x/b$, $\eta = y/b$, and $\zeta = z/b$, while b is a suitable length used for purposes of nondimensionalization. The value used for b will be equal to the semispan of the wing in the case where the leading edge of the wing is supersonic everywhere, and provided that the wing tips are cut off in such a way that the wing surface remains outside of the tip Mach cones emanating from either one of the wing-tip extremities out at the farthest reaches of the wing span. The value used for b will be larger than this semispan just defined, if, in contrast, these geometrical relationships do not hold; the magnitude employed for b in this latter case is illustrated in figure 2.

Finally, it should be observed that H^* is a periodic function of y, which is equal to the values taken on by the function H at the wing's surface and it is zero for points lying out of this region, and this definition is to hold throughout the spanwise interval for which $-b \leq y \leq b$.

The fact that it is possible to write $H^*(x,y)$ in the form given as equation (4) (i.e., the possibility of expressing the component-velocity field describing the normal velocities to the wing surface by means of a Fourier series instead of in terms of a Fourier integral) stems from the property already noted to the effect that the perturbations, which are created at any arbitrary point $P(x,y)$ whatsoever, do not make themselves felt anywhere outside of the Mach cone emanating from P. As a result of this situation, therefore, as far as the

determination of the field of flow about the given wing is concerned, it makes no difference to this flow whether one considers the wing to be operating by itself as an isolated entity within the impinging stream or whether, instead, one imagines it to be accompanied by an infinite number of reflections of this primary wing in the planes $y = \pm mb$.

If one now inserts the second of the expressions given as equation (4) into equation (1), it will be seen that this differential equation reduces to

$$\frac{\partial^2 \phi_m}{\partial \zeta^2} - B^2 \frac{\partial^2 \phi_m}{\partial \xi^2} = k^2 \phi_m \tag{5}$$

wherein $B^2 = M_\infty^2 - 1$ and where k replaces the constant $\frac{\pi m}{2}$.

Meanwhile, it is also evident that, on the basis of the first of the formal developments given as equation (4), the boundary condition reduces to

$$\left(\frac{\partial \phi_m}{\partial \zeta}\right)_{\zeta=0} = H_m(\xi) \tag{6}$$

The expression given as equation (5) above is formally analogous to the so-called "telegraph equation," and its solution, which is suitable for applying the type of boundary condition exemplified in equation (6), is

$$\phi_m = \frac{\pi}{B} \int_0^{\xi - B\zeta} h_m(\xi') \, J_0 \left[\frac{k}{B} \sqrt{(\xi-\xi')^2 - B^2\zeta^2}\right] d\xi' \tag{7}$$

where J_0 is the cylindrical Bessel function of zeroth order.

Consequently, the vertical derivative turns out to be

$$\frac{\partial \phi_m}{\partial \zeta} = \pi h_m(\xi - B\zeta) + \pi k\zeta \int_0^{\xi - B\zeta} h_m(\xi') \, J_1 \left[\frac{k}{B} \sqrt{(\xi-\xi')^2 - B^2\zeta^2}\right] \frac{d\xi'}{\sqrt{(\xi-\xi')^2 - B^2\zeta^2}}$$

and, because of the boundary condition (6), it follows that

$$h_m = -\frac{1}{\pi} H_m$$

so that the sought potential must have the form

$$\phi_m = -\frac{1}{B} \int_0^{\xi - B\zeta} H_m(\xi') J_0 \, d\xi' \qquad (7')$$

4. DEVELOPMENT OF THE CASE OF THE ASYMMETRIC TYPE OF CONFIGURATION

The possibility of being able to find solutions to such asymmetric problems by means of the method being propounded here is restricted in this case to those configurations for which the leading edge as well as the trailing edge of the wing are supersonic, and where the wing tips are cut off in such a way that the wing surface lies outside of the Mach cone emanating from the very tip of the leading edge where the maximum span occurs.

Under these circumstances the boundary conditions are constituted from the restrictions given as equations (3"), and of equation (2') once again. If one then follows the same procedure as was utilized in section 3, it follows that the expression for the sought potential is formally given as (ref. 1)

$$\phi_m = \pm \pi h_m \left[\xi - B|\zeta| \right] + \pi k \zeta \int_0^{\xi - B|\zeta|} h_m(\xi') J_1 \left[\frac{k}{B} \sqrt{(\xi - \xi')^2 - B^2 \zeta^2} \right] \frac{d\xi'}{\sqrt{(\xi - \xi')^2 - B^2 \zeta^2}}$$

$$(8)$$

where h_m is, _a priori_, an undetermined function, and where it should be recognized that the + sign is to be employed for the lower half-plane where $\zeta < 0$, and where the - sign is to be employed for the upper half-plane where $\zeta > 0$. It is evident, therefore, that the derivative of ϕ_m with respect to ζ will be continuous along the plane $\zeta = 0$, but the

derivative of ϕ_m with respect to ξ will be discontinuous, and the "jump" will be of such size that

$$\left(\frac{\partial \phi_m}{\partial \xi}\right)_{\zeta=0^+} = -\left(\frac{\partial \phi_m}{\partial \xi}\right)_{\zeta=0^-}$$

holds true.

It is clearly permissible here again to concentrate attention solely upon the disturbed flow in the upper half-plane where $\zeta > 0$, therefore, because the observation just made above will tell one how to compute what the flow will be in the other lower half-plane, once the former is obtained.

The boundary conditions in this instance may now be recast into the form

$$\pi B \dot{h}_m(\xi) + \pi \frac{k^2}{B} \int_0^\xi h_m(\xi') \ J_0 \left(k \ \frac{\xi - \xi'}{B}\right) d\xi' -$$

$$\pi k \int_0^\xi \dot{h}_m(\xi') \ J_1 \left(k \ \frac{\xi - \xi'}{B}\right) d\xi' = G_m(\xi)$$

provided, as in the previous section, one sets up the convenient convention that $G^*(x,y)$ is to represent a periodic function in y that is to be equal to the values taken on by the function $G(x,y)$ at the wing's surface, and it is to be zero for points lying out of this region. This definition is to hold throughout the spanwise interval for which $-b \leq y \leq b$. In addition, the form of $G^*(x,y)$ is to be assumed, specifically, to have the appearance

$$G^*(x,y) = V_\infty \sum_m G_m(\xi) \ \cos \frac{\pi}{2} m\eta$$

while it has also been assumed that the derivative of a function by the sole parameter upon which it depends is to be denoted by a dot over the function, that is,

$$\dot{h}_m(\xi) = \frac{dh_m}{d\xi}$$

The integro-differential equation defining h_m may also be immediately simplified to the compressed expression

$$\pi B \dot{h}_m(\xi) + \pi \frac{k^2}{2B} \int_0^\xi h_m(\xi') \left(J_0 + J_2\right) d\xi' = G_m(\xi) \qquad (9)$$

Now apply a Laplace transformation to this integro-differential equation $\left(\text{i.e., multiply through by the factor } e^{-p\xi} \text{ and integrate from 0 to } \infty\right)$. Thus, one obtains

$$\pi B \overline{h}_m p + \pi \frac{k^2}{2B} \overline{h}_m \left[\frac{1}{\sqrt{p^2 + \frac{k^2}{B^2}}} + \frac{\left(\sqrt{p^2 + \frac{k^2}{B^2}} - p\right)^2}{\frac{k^2}{B^2}\sqrt{p^2 + \frac{k^2}{B^2}}} \right] = \overline{G}_m$$

where a bar over a symbol serves to indicate that this quantity stands for the Laplace transform of the function so designated.

Standard tables of Laplace transforms could be consulted to check these results, which may now be simplified by noting that

$$\frac{1}{\sqrt{p^2 + \frac{k^2}{B^2}}} + \frac{\left(\sqrt{p^2 + \frac{k^2}{B^2}} - p\right)^2}{\frac{k^2}{B^2}\sqrt{p^2 + \frac{k^2}{B^2}}} = \frac{\frac{k^2}{B^2} + p^2 + \frac{k^2}{B^2} + p^2 - 2p\sqrt{p^2 + \frac{k^2}{B^2}}}{\frac{k^2}{B^2}\sqrt{p^2 + \frac{k^2}{B^2}}}$$

$$= 2 \frac{B^2}{k^2}\left(\sqrt{p^2 + \frac{k^2}{B^2}} - p\right)$$

Thus the Laplace transforms of equation (9) simplifies to

$$\pi B p \overline{h}_m + \pi B \left(\sqrt{p^2 + \frac{k^2}{B^2}} - p\right) \overline{h}_m = \overline{G}_m$$

or the explicit expression for the Laplace transform of the unknown function h_m is given in the form

$$\overline{h}_m = \frac{\overline{G}_m}{\pi B} \frac{1}{\sqrt{p^2 + \frac{k^2}{B^2}}}$$

so that finally one may invert the transformation to obtain

$$h_m = \frac{1}{\pi B} \int_0^\xi G_m(\xi')J_0\left[\frac{k}{B}(\xi - \xi')\right] d\xi' \qquad (10)$$

Once having obtained the value of h_m, it is easy to write down the expression for the component of velocity lying in the x-direction and located at the wing-surface, because one has simply that this component is given by the partial derivative of the potential ϕ, taken with respect to ξ, and evaluated at the plane of the wing; i.e., one finds that

$$\left(\frac{\partial \phi}{\partial \xi}\right)_{\zeta=0} = V_\infty \sum_m \left(\frac{\partial \phi_m}{\partial \xi}\right)_{\zeta=0} \cos \frac{\pi}{2} m\eta = -V_\infty \pi \sum_m \dot{h}_m(\xi) \cos \frac{\pi}{2} m\eta$$

Furthermore, the formula giving the lift on the wing is just

$$L = 8\rho_\infty V_\infty^2 b^2 \sum_m (-1)^{\frac{m+1}{2}} \frac{h_m(l/b)}{m} \qquad (11)$$

$$\text{for } m = 1,3,5,\ldots, \text{ etc.}$$

where the symbol, l, is used to denote the distance along the x-axis measured back from the leading edge of the root chord to the projection into the plane of symmetry of the trailing edge of the wing-tip profile.

In regard to the moment taken about the y-axis, it is apparent that it may be computed from the relation:

$$M_y = 2\pi\rho_\infty V_\infty^2 b^3 \sum_m \int_{-1}^1 \cos\left(\pi m \frac{\eta}{2}\right) d\eta \int_0^{l/b} \xi \dot{h}_m(\xi) d\xi$$

$$= 2\pi\rho_\infty b^3 V_\infty^2 \sum_m \left(\frac{-4}{m\pi}\right)(-1)^{\frac{m+1}{2}} \left[\frac{l}{b} h_m\left(\frac{l}{b}\right) - \int_0^{l/b} h_m(\xi) d\xi\right] \qquad (12)$$

$$\text{for } m = 1,3,5,\ldots, \text{ etc.}$$

5. PROBLEMS HAVING TO DO WITH FLOW PAST BODIES OF REVOLUTION

The procedure discussed in the preceding sections can be extended at once to apply also to the solution of problems which are concerned with the flow over bodies of revolution.

For this purpose let a cylindrical coordinate system (x, Y, θ) be set up, and then the equation which governs the potential, ϕ, being sought will have the form

$$\frac{\partial^2 \phi}{\partial Y^2} + \frac{1}{Y}\frac{\partial \phi}{\partial Y} + \frac{1}{Y^2}\frac{\partial^2 \phi}{\partial \theta^2} = B^2 \cdot \frac{\partial^2 \phi}{\partial x^2} \tag{13}$$

Now let $R = R(x)$ be the equation of the meridian line of the body, and let it be assumed that R is sufficiently small at all locations along the body so that the direction cosine of the normal to this meridian line, measured from the Y-axis, may be taken to be equal to unity.

Furthermore, let V_{nf}' stand for the component, taken in a direction perpendicular to the circular cross-section of the body, which arises from the impinging flow which invests the body. Then the boundary condition which must be satisfied at the surface of the body may be expressed mathematically by the relation

$$\left(\frac{\partial \phi}{\partial Y}\right)_{Y=R} = -V_{nf}'$$

For sake of simplicity, it is also now assumed that the treatment to be developed is to be restricted to the case where symmetry with respect to the semiplanes $\theta = \pm\frac{\pi}{2}$ exists in the incident flow. Under this hypothesis it is convenient to write the normal velocity components and the potential being sought in the following explicit formulations:

$$\left.\begin{aligned} V_{nf}' &= V_\infty \sum_m F_m\left(\frac{x}{R}\right) \sin m\,\theta \\[2ex] \phi &= V_\infty \sum_m \phi_m(x,Y)Y^m \sin m\,\theta \end{aligned}\right\} \tag{14}$$

If one now inserts the second of the expressions given as equation (14) into the differential equation governing the flow (13), it will be found that the defining equation for the potential will have the form

$$\frac{\partial^2 \phi_m}{\partial Y^2} + \frac{2m + 1}{Y} \frac{\partial \phi_m}{\partial Y} = B^2 \frac{\partial^2 \phi_m}{\partial x^2} \tag{15}$$

and the boundary condition turns out to be

$$\frac{\partial \left(Y^m \phi_m \right)}{\partial Y} \bigg|_{Y=R} = -F_m \tag{15'}$$

A suitable solution to equation (15), which can be made to satisfy the boundary condition being imposed as equation (15'), will be found to be

$$\phi_m = \left(\frac{1}{Y} \frac{\partial}{\partial Y} \right)^m \phi_0 \tag{16}$$

where

$$\phi_0 = \int_{\text{arc cosh} \frac{x}{BY}}^{0} f_0(x - BY \cosh u) du$$

Thus, the successive individual potentials are given by the expressions[1]

$$Y\phi_1 = -B \int_{\text{arc cosh } \frac{x}{BY}}^{0} f_1(x\text{-BY cosh } u) \cosh u \, du$$

$$Y\phi_2 = B^2 \int_{\text{arc cosh } \frac{x}{BY}}^{0} f_2(x\text{-BY cosh } u) \cosh^2 u \, du \qquad (17)$$

etc.

Upon imposition of the requirement that the boundary condition (15') is to be satisfied, one obtains a set of integral equations which serve

---

[1]Translator's note: It was pointed out on page 630 of an article by R. H. Cramer in the Journal of the Aeronautical Sciences, vol. 18, no. 9, September 1951, entitled "Interference Between Wing and Body at Supersonic Speeds - Theoretical and Experimental Determination of Pressures on the Body," that the result given here for ϕ_m, for $m > 1$, is incorrect; the correct formula is, for $m = 2$,

$$Y^2\phi_2 = B^2 \int_{\text{arc cosh } \frac{x}{BY}}^{0} f_2(x\text{-BY cosh } u)(2\cosh^2 u - 1) \, du$$

while, in general, the use of hyperbolic functions of multiples of the argument u gives a more compact form, which is easy to work with; i.e., in general it is true that

$$Y^m\phi_m = B^m(-1)^m \int_{\text{arc cosh } \frac{x}{BY}}^{0} f_m(x\text{-BY cosh } u)(\cosh m u) \, du$$

to determine the arbitrary functions f_m, which are <u>a priori</u> unknown.
Thus, applying these conditions, one finds that[2]

$$B^{m+1} (-1)^{m+1} \int_{\text{arc cosh } \frac{x}{BR}}^{0} \dot{f}_m(x-BR \cosh u) (\cosh u)^{m+1} du = -F_m \quad (18)$$

The determination of the values of the $\dot{f}_m$'s appearing in formula (18)
may be carried out by using a step-by-step procedure which is entirely
analogous to the one employed by Von Kármán in his work on determining
the flow about a body of revolution at zero angle of attack.

It is important to point out that if one only has in mind to calcu-
late the force distribution along the axis of the body and the corre-
sponding moment, and if one is not interested in knowing the local veloci-
ties or pressures around the body, then it is merely necessary to
calculate $\dot{\phi}_0$ and $\dot{\phi}_2$.

6. APPLICATIONS

The procedure that has been propounded above has been applied (ref. 2)
to the situation arising in the study of the question of wing-body inter-
ference. The wing-body configuration considered in this particular appli-
cation of the method is depicted in the appended figure 3.

The wing used in this configuration is a flat plate, whose semispan
is equal to $4R_0$, where R_0 is the radius of the circular cross-section
taken through the body at the location where the body is widest. The
leading edge of this wing is located $5R_0$ downstream from the tip of the
nose of the body. The free-stream flow is impinging on the body at a
speed which is twice the speed of sound in the undisturbed stream.

---

[2]Translator's note: In view of the correction pointed out in Note 1
above, it will be seen that this formula for determining the f_m functions
is also incorrect, except for $m = 1$; for higher integral values of m,
the correct formula is:

$$B^{m+1} (-1)^{m+1} \int_{\text{arc cosh } \frac{x}{BR}}^{0} \dot{f}_m(x-BR \cosh u) \left[\cosh m u \cosh u\right] du = -F_m$$

The curves shown in figure 3 give the value of the pressure coeffi-
cient, $\dfrac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$, out along the span of the wing, in the mid-chord loca-
tion (i.e., along the wing axis), for points on the upper (dorsal) side
of the wing. These coefficients have been calculated by the method out-
lined in section 4, and there are shown results for various angles of
attack, which apply to such points on the upper side of the wing pro-
files at their mid-chord positions.

In addition, some experimental test points obtained by R. H. Cramer
(see ref. 2) are also plotted on these curves. These results were
obtained from experiments carried out in the supersonic tunnel of the
Daingerfield Aeronautical Laboratory.

The agreement between the computed and experimentally determined
results is very good from a qualitative viewpoint. In regard to the
more precise details of the quantitative comparison between the results
it is worthy of note that the experimental results exhibit a certain
amount of dissymmetry as one passes from positive angles of attack to
negative angles of attack. Such a dissymmetry cannot be predicted, or
should not be expected, from the type of theoretical treatment being
considered here.

In order to bring about a more valid comparison of these results,
it would appear logical, in face of such evident dissymmetry, to take
for the representative experimental value, at a given value of the angle
of attack, β, the one which is obtained from averaging the result
obtained for an angle of attack equal to $+\beta$ with the result obtained
at $-\beta$. Such average values have been computed and are designated in
the plots of figure 3 by means of solid circles. These adjusted values
lie much closer to the theoretically derived curves at almost all
locations.

Translated by R. H. Cramer
Cornell Aeronautical Laboratory, Inc.
Buffalo, New York

REFERENCES

1. Ferrari, C.: Interference Between Wing and Body at Supersonic Speeds -
    Theory and Numerical Application. Jour. Aero. Sci., vol. 15, no. 6,
    June 1948, pp. 317-336.

2. Ferrari, C.: Interference Between Wing and Body at Supersonic Speeds -
    Note on Wind Tunnel Results and Addendum to Calculations. Jour.
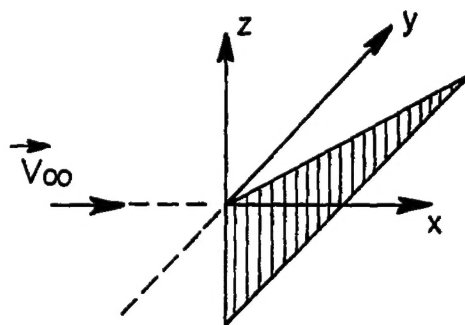   Aero. Sci., vol. 16, no. 9, Sept. 1949, pp. 542-546.

Figure 1.- Orientation of coordinate axes and location of typical wing plan form therein.
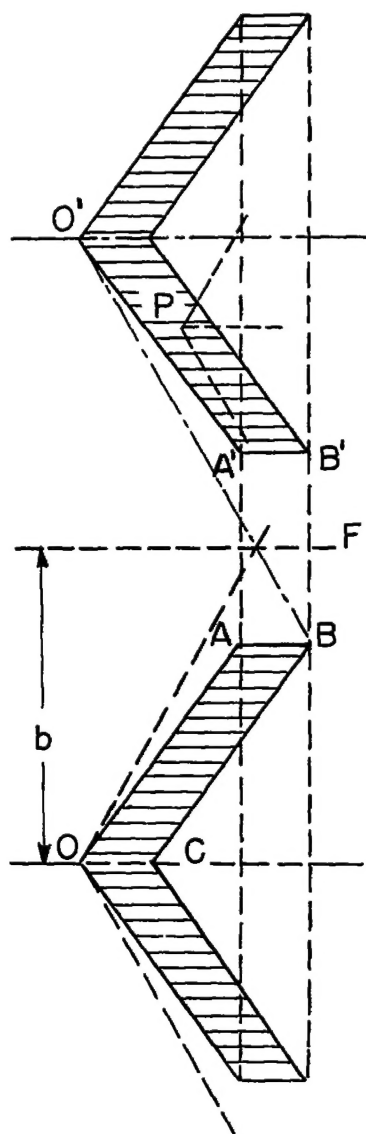

Figure 2.- Definition of the interval of periodicity required for application of the Fourier series technique when leading edges are subsonic.

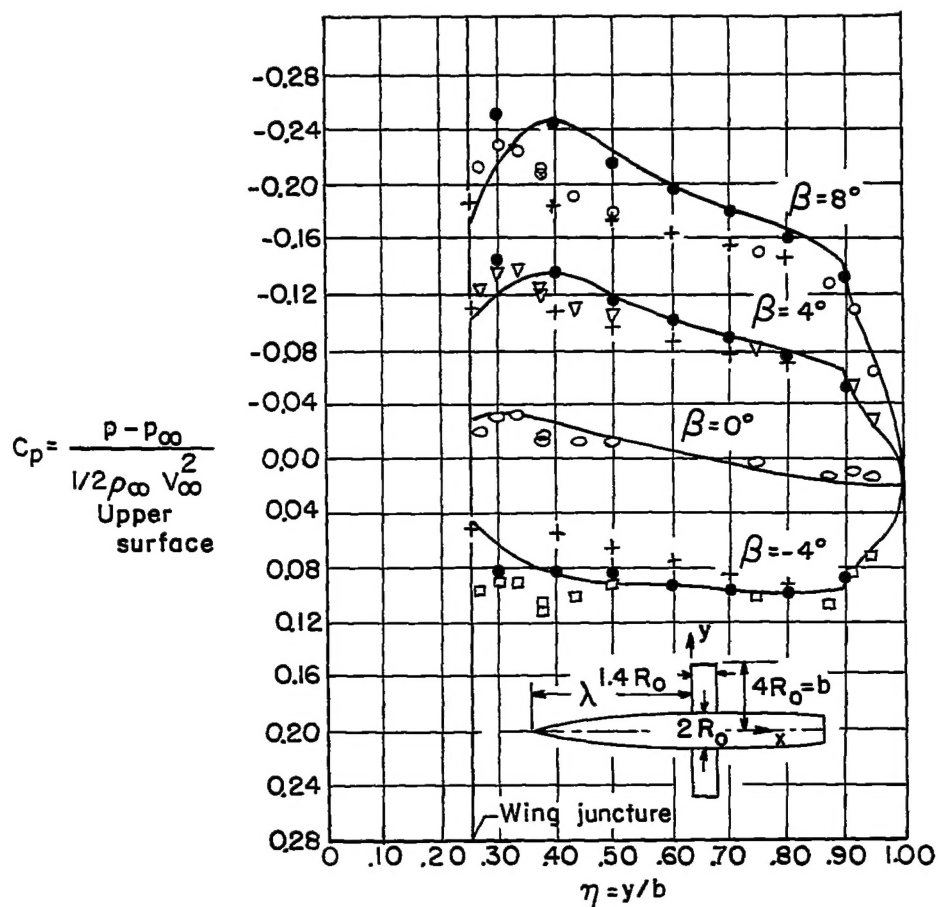$$C_p = \frac{p - p_\infty}{1/2 \rho_\infty V_\infty^2}$$
Upper surface

Figure 3.- Pressure distribution along the wing axis: Comparison of experimental results with predictions based on the method expounded in section 4.